## Desperately seeking software perfection

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Inria Paris-Rocquencourt, projet Gallium

Colloquium d'informatique, UPMC, octobre 2015



## Part I

Imperfect software

#### Software crashes...



Paris highway



Las Vegas billboard

#### Software crashes...



Metro station, Manhattan



Heathrow airport

#### Software crashes...



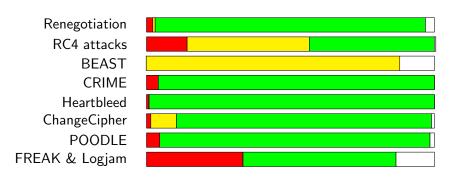
Olympic games, 2008



Nine Inch Nails concert

#### Software has security holes...

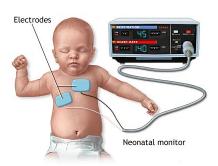
Major Web security issues (TLS protocol, Web servers) since 2009:





#### Software kills...





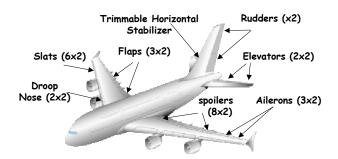
Therac 25 radiation machine (3 patients dead following massive overdose.)

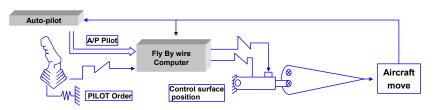
Newborn monitor (several cases of sudden infant death where the alarm did not ring)

#### Part II

A glimpse of hope: Critical avionics software

#### Running example: fly-by-wire software





#### **Timeline**



#### Functions of FBW software

High AOA	Load Factor	Pitch Attitude		
Protection	Limitation	Protection		
NORMAL LAW				
High Speed	Flight Augmentation	Bank Angle		
Protection	(Yaw)	Protection		

Low Speed Stability	Load Factor Limitation			
ALTERNATE LAW				
High Speed Stability	Yaw Damping Only			

	Load Factor Limitation			
ABNORMAL ALTERNATE LAW w/o Speed Stability				
	Yaw Damping Only			

DIRECT LAW			

Execute pilot's commands.

Flight assistance: keep aircraft within safe flight envelope.

Fuel economy: minimize drag.

Active damping of oscillations.

#### Anatomy of FBW systems

#### Two-part software:

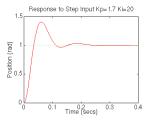
- A minimalistic operating system
   (Boot, self-tests, communications over buses, static scheduling of periodic tasks. Generally hand-crafted, sometimes off-the-shelf.)
- Mostly: control-command code
   (Simulink, Scade)
   (≈ discretized differential equations)

Hard real-time.

100k – 1M LOC of C code, but mostly generated from Scade / Simulink.

Asymmetric redundancy (e.g. 3 primary units, 3 secondary).

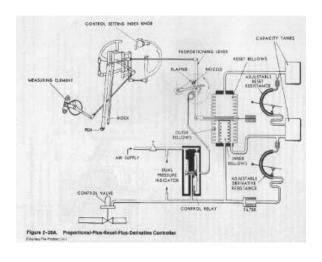
"Hello, world" example: PID controller.



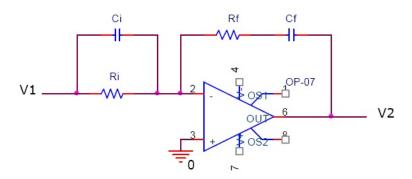
Error e(t) = desired state(t) - current state(t).

Action 
$$a(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{d}{dt} e(t)$$
  
(Proportional) (Integral) (Derivative)

#### Mechanical (e.g. pneumatic):



#### Analog electronics:



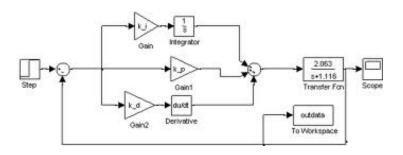
```
In software (today's favorite solution):

previous_error = 0; integral = 0
loop forever:
    error = setpoint - actual_position
    integral = integral + error * dt
    derivative = (error - previous_error) / dt
    output = Kp * error + Ki * integral + Kd * derivative
    previous_error = error
    wait(dt)
```

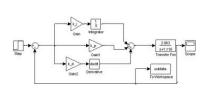
#### Block diagrams

(Simulink, Scade, Scicos, etc)

This kind of code is rarely hand-written, but rather auto-generated from block diagrams:



## Block diagrams and reactive languages







(P. Caspi)

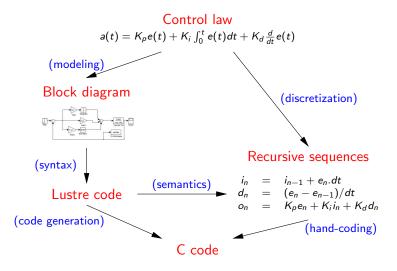
(N. Halbwachs)

In the case of Scade, this diagram is a graphical syntax for the Lustre reactive language:

```
error = setpoint - position
integral = (0 -> pre(integral)) + error * dt
derivative = (error - (0 -> pre(error))) / dt
output = Kp * error + Ki * integral + Kd * derivative
```

(= Time-indexed series defined by recursive equations.)

#### Block diagrams and reactive languages



## A successful Domain Specific Language

- "Speaks" the language of users who are not programmers. (Pseudo-circuits in graphical syntax.)
- Supports automatic generation of efficient code. (The model is the program.)
- Reduced expressiveness.
   (A language of boxes, wires, latches and clocks; Turing-incomplete)
- Supports formal verification.

## The certification process (DO-178)



Design and development process is meticulous and fully documented.

Rigorous validation at multiple levels (from design to product):

- Reviews (qualitative)
- Analyses (quantitative)
- Test, test!, test!!, test, test, test, test, ...
- Recent development: use of formal verification tools.

#### From unit testing...

```
double max(double x, double y)
{
    if (x >= y) return x; else return y;
}
```

```
\max(0,0) = 0 \max(1,-1) = 1 \max(0,1) = 1 \max(0,-1) = 0 \max(1,\inf) = \inf \max(0,3.14) = 3.14 \max(0,\inf) = \inf \max(0,\inf) = \inf \max(0,\inf) = \inf \max(0,\inf) = \inf \max(0,-\inf) = 0 \max(0,n) = 0 \max(1,0) = 1 \max(1,1) = 1
```

## ... to integration testing...



## ... to exploration on an Iron Bird...





# ... to test flights









Tool-assisted formal verification

Part III

## Beyond testing: formal verification

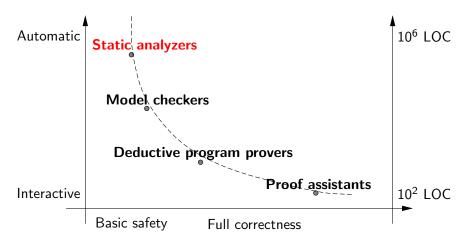
Program testing can be used to show the presence of bugs, but never to show their absence!

(E.W.Dijkstra, 1972)

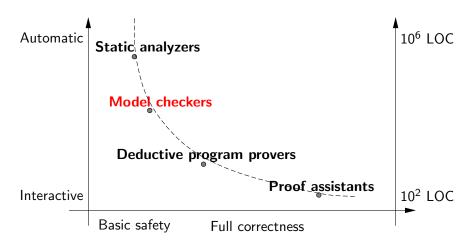
Formal verification of software: verify, possibly infer, properties that hold of all possible executions of a program.

Used in some industrial contexts (airplanes, railways)

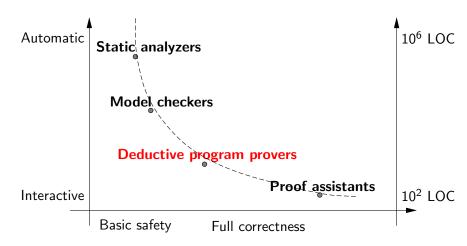
- To obtain independent guarantees (besides testing).
- To obtain stronger guarantees (than with testing).
- To replace costly unit tests.



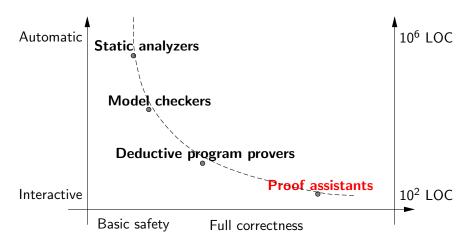
Static analysis: automatically infer simple properties of one variable  $(x \in [N_1, N_2], x \mod N = 0, \text{ etc})$  or several  $(x + y \le z)$ .



Model checking: automatically check that some "bad" program points are not reachable.



Program proof: show that  $preconditions \Rightarrow invariants \Rightarrow postconditions$  using automated theorem provers.



Proof assistants: conduct mathematical proofs in interaction with the user; re-check the proofs for correctness.

```
int a[] = new int[n];
a[0] = 2;
loop:
for (int i = 1, m = 3; i < n; m = m + 2) {
   int j = 0;
   while (j < i \land a[j] <= \sqrt{m}) {
      if (a[j] divides m) continue loop;
      j = j + 1;
   }
   a[i] = m; i = i + 1;
}
```

Goal: compute the first n prime numbers.

Algorithm: try successive odd numbers m, striking out those divisible by primes already found.

```
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        j = j + 1;
    }
    a[i] = m; i = i + 1;
}
```

Static analyzer: can infer  $1 \le i < n$  and  $0 \le j < i$  inside the loop, hence array accesses are safe (within bounds).

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        j = j + 1;
    }
    a[i] = m; i = i + 1;
}
```

Automatic program prover: can prove partial correctness if the user provides detailed loop invariants and simple axioms about primality and divisibility. (Termination is harder to prove.)

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Knuth, The Art of Computer Programming, vol.1

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   }
}
```

Knuth's cunning optimization: the test j < i is redundant and can be omitted. Can you see why? Because of Bertrand's postulate!

```
Theorem (Chebychev)
```

For all  $n \ge 1$ , there exists a prime p in [n, 2n].

(Coq proof: Laurent Théry, 2002.)

### Example: computing prime numbers

Knuth, The Art of Computer Programming, vol.1

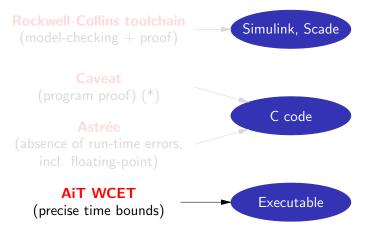
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      j = j + 1;
   }
```

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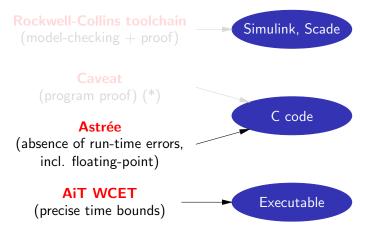
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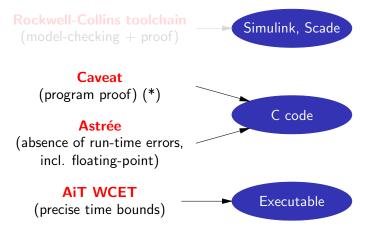
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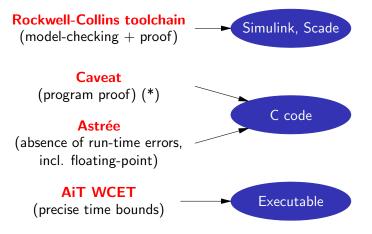
<sup>(</sup>st) Motto: ''unit proofs as a replacement for unit tests'



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# Success stories in verification of systems code

#### The seL4 secure microkernel: (NICTA, 2009)

- Full correctness proof of a high-performance microkernel.
- Using the Isabelle/HOL proof assistant + custom automation.
- 8 KLOC of C code, 200 KLOC proof, 20 person.years.
- The largest deductive verification of a software system ever.

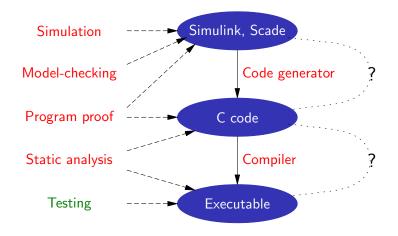
#### The FSCQ file system: (MIT, 2015)

- Formally proved correct even in the presence of crashes.
- Using the Coq proof assistant + custom automation.
- 30 KLOC proof, 1.5 person.years.

# Part IV

Formally-verified compilation

#### Trust in software verification



The unsoundness risk: Are verification tools semantically sound? The miscompilation risk: Are compilers semantics-preserving?

## Miscompilation happens

NULLSTONE isolated defects [in integer division] in twelve of twenty commercially available compilers that were evaluated.

http://www.nullstone.com/htmls/category/divide.htm

We tested thirteen production-quality C compilers and, for each, found situations in which the compiler generated incorrect code for accessing volatile variables.

E. Eide & J. Regehr, EMSOFT 2008

To improve the quality of C compilers, we created Csmith, a randomized test-case generation tool, and spent three years using it to find compiler bugs. During this period we reported more than 325 previously unknown bugs to compiler developers. Every compiler we tested was found to crash and also to silently generate wrong code when presented with valid input.

X. Yang, Y. Chen, E. Eide & J. Regehr, PLDI 2011

### An example of optimizing compilation

```
double dotproduct(int n, double * a, double * b)
{
    double dp = 0.0;
    int i;
    for (i = 0; i < n; i++) dp += a[i] * b[i];
    return dp;
}</pre>
```

Compiled with a good compiler, then manually decompiled back to  $\mathsf{C}\dots$ 

```
double dotproduct(int n, double a[], double b[]) {
    dp = 0.0;
    if (n <= 0) goto L5:
     r2 = n - 3: f1 = 0.0: r1 = 0: f10 = 0.0: f11 = 0.0:
     if (r2 > n || r2 <= 0) goto L19;
     prefetch(a[16]): prefetch(b[16]):
     if (4 \ge r2) goto L14:
     prefetch(a[20]); prefetch(b[20]);
    f12 = a[0]; f13 = b[0]; f14 = a[1]; f15 = b[1];
     r1 = 8; if (8 \ge r2) goto L16;
L17: f16 = b[2]; f18 = a[2]; f17 = f12 * f13;
     f19 = b[3]; f20 = a[3]; f15 = f14 * f15;
    f12 = a[4]: f16 = f18 * f16:
    f19 = f29 * f19; f13 = b[4]; a += 4; f14 = a[1];
    f11 += f17; r1 += 4; f10 += f15;
    f15 = b[5]: prefetch(a[20]): prefetch(b[24]):
    f1 += f16: dp += f19: b += 4:
     if (r1 < r2) goto L17;
L16: f15 = f14 * f15; f21 = b[2]; f23 = a[2]; f22 = f12 * f13;
     f24 = b[3]: f25 = a[3]: f21 = f23 * f21:
    f12 = a[4]; f13 = b[4]; f24 = f25 * f24; f10 = f10 + f15;
     a += 4; b += 4; f14 = a[8]; f15 = b[8];
    f11 += f22; f1 += f21; dp += f24;
L18: f26 = b[2]: f27 = a[2]: f14 = f14 * f15:
    f28 = b[3]; f29 = a[3]; f12 = f12 * f13; f26 = f27 * f26;
     a += 4: f28 = f29 * f28: b += 4:
     f10 += f14: f11 += f12: f1 += f26:
     dp += f28; dp += f1; dp += f10; dp += f11;
     if (r1 \ge n) goto L5;
L19: f30 = a[0]: f18 = b[0]: r1 += 1: a += 8: f18 = f30 * f18: b += 8:
     dp += f18;
    if (r1 < n) goto L19;
L5: return dp:
L14: f12 = a[0]; f13 = b[0]; f14 = a[1]; f15 = b[1]; goto L18;
```

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     dp += f28; dp += f1; dp += f10; dp += f11;
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L14: f12 = a[0]; f13 = b[0]; f14 = a[1]; f15 = b[1]; goto L18;
```

## Addressing miscompilation

Best industrial practices: more testing; manual reviews of generated assembly code; turn optimizations off; ...

A more radical solution: why not formally verify the compiler itself?

After all, compilers have simple specifications:

If compilation succeeds, the generated code should behave as prescribed by the semantics of the source program.

As a corollary, we obtain:

Any safety property of the observable behavior of the source program carries over to the generated executable code.

#### An old idea...

John McCarthy
James Painter

1

# CORRECTNESS OF A COMPILER FOR ARITHMETIC EXPRESSIONS<sup>2</sup>

 Introduction. This paper contains a proof of the correctness of a simple compiling algorithm for compiling arithmetic expressions into machine language.

The definition of correctness, the formalism used to express the description of source language, object language and compiler, and the methods of proof are all intended to serve as prototypes for the more complicated task of proving the correctness of usable compilers. The ultimate goal, as outlined in references [1], [2], [3] and [4] is to make it possible to use a computer to check proofs that compilers are correct.

Mathematical Aspects of Computer Science, 1967

3

# Proving Compiler Correctness in a Mechanized Logic

R. Milner and R. Weyhrauch Computer Science Department

Stanford University

#### Abstract

We discuss the task of machine-checking the proof of a simple compiling algorithm. The proof-checking program is LCF, an implementation of a logic for computable functions due to Dana Scott, in which the abstract syntax and extensional semantics of programming languages can be naturally expressed. The source language in our example is a simple ALGOL-like language with assignments, conditionals, whiles and compound statements. The target language is an assembly language for a machine with a pushdown store. Algebraic methods are used to give structure to the proof, which is presented only in outline. However, we present in full the expression-compiling part of the algorithm. More than half of the complete proof has been machine checked, and we anticipate no difficulty with the remainder. We discuss our experience in conducting the proof, which indicates that a large part of it may be automated to reduce the human contribution.

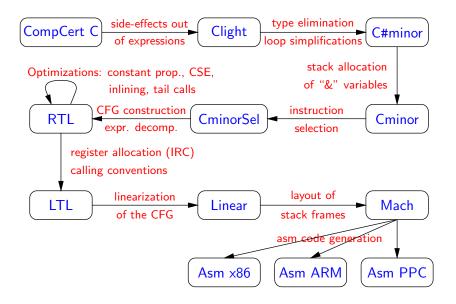
# The CompCert project (X.Leroy, S.Blazy, et al)

Develop and prove correct a realistic compiler, usable for critical embedded software.

- Source language: a very large subset of C99.
- Target language: PowerPC/ARM/x86 assembly.
- Generates reasonably compact and fast code
  - $\Rightarrow$  careful code generation; some optimizations.

Note: compiler written from scratch, along with its proof; not trying to prove an existing compiler.

## The formally verified part of the compiler



## Formally verified using Coq

The correctness proof (semantic preservation) for the compiler is entirely machine-checked, using the Coq proof assistant.

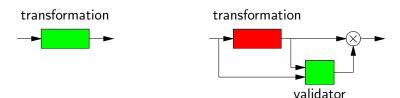
#### Shows refinement of observable behaviors beh:

- Reduction of internal nondeterminism
   (e.g. choose one evaluation order among the several allowed by C)
- Replacement of run-time errors by more defined behaviors (e.g. optimize away a division by zero)

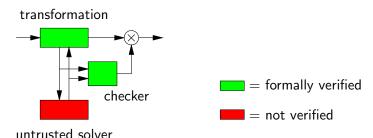
#### Compiler verification patterns (for each pass)

#### Verified transformation

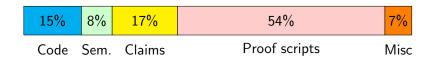
#### Verified translation validation



#### External solver with verified validation



#### Proof effort



100,000 lines of Coq.

Including 15000 lines of "source code" ( $\approx$  60,000 lines of Java).

6 person.years

Low proof automation (could be improved).

## Programmed (mostly) in Coq

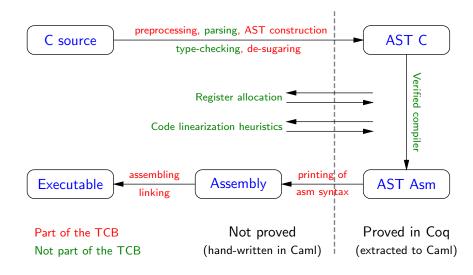
All the verified parts of the compiler are programmed directly in Coq's specification language, using pure functional style.

- Monads to handle errors and mutable state.
- Purely functional data structures.

Coq's extraction mechanism produces executable Caml code from these specifications.

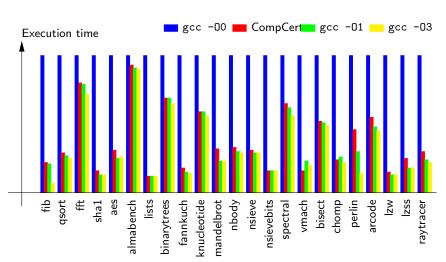
Claim: purely functional programming is the shortest path to writing and proving a program.

### The whole Compcert compiler



## Performance of generated code

(On a Power 7 processor)



### A tangible increase in quality

The striking thing about our CompCert results is that the middleend bugs we found in all other compilers are absent. As of early 2011, the under-development version of CompCert is the only compiler we have tested for which Csmith cannot find wrong-code errors. This is not for lack of trying: we have devoted about six CPU-years to the task. The apparent unbreakability of CompCert supports a strong argument that developing compiler optimizations within a proof framework, where safety checks are explicit and machine-checked, has tangible benefits for compiler users.

X. Yang, Y. Chen, E. Eide, J. Regehr, PLDI 2011

## Part V

# Conclusions

### Is software perfection within reach?

#### Perhaps. But at a minimum we need:

• ... and further research!

<ul> <li>Mathematical specifications</li> </ul>	(e.g. control-command)
<ul> <li>Appropriate programming languages</li> </ul>	(e.g. Scade)
<ul> <li>Serious testing</li> </ul>	(of the aircraft kind)
<ul> <li>Formal verification</li> </ul>	(Astrée, CADP, Frama-C, )
<ul> <li>Trustworthy tools</li> </ul>	(CompCert, Verasco)
• Theorem proving	(Cog. 73 Alt-Frgo